Hodographic Prediction of Cyclic Creep Behavior

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Experimental study of the effect of discontinuous changes in strain-rate on the relationship between strain-rate, strain, and stress is described. Data from Udimet 700 in tension at 925C are used to relate cyclic tensile creep to the monotonic properties of the material by means of the hodograph, a plot of strain-rate against strain. The nature of modifications caused to the hodograph by discontinuous variation of the strain-rate is determined from tests. Reloading at discontinuous strain-rate causes reactivation of primary creep. A simple method is proposed for predicting cyclic tensile creep response. Results of cyclic tests agree with predicted response.

Nomenclature

E = modulus of elasticity
t = time
ϵ = true strain
$\dot{\epsilon}$ = strain-rate
σ = true stress

subscripts

f = fracture
i = inelastic
$\Delta\sigma$ = stress change

Introduction

A MAJOR problem in predicting low-cycle fatigue at elevated temperatures has been that of estimating the effect on life of deformation processes which are strongly rate-dependent. These include creep while strain is either increasing or decreasing in the fatigue cycle, and stress relaxation while the strain is held constant (Fig. 1). Success in estimating low-cycle fatigue life under such conditions has varied considerably (see bibliography in Ref. 1). Methods of analysis include the 10% rule (Ref. 2) and the separate calculation and subsequent summation of the time-independent (fatigue) damage and the time-dependent (creep) damage.[3,4] More recently a strain-partitioning approach has been suggested whereby the cyclic strain increment is divided into its constituent components of time-independent and time-dependent deformation.[1]

A fundamental variable, which has not been fully explored, but whose contribution is considered significant, is the strain-rate, related to the cyclic frequency. Strain-rate has been shown[5] to be the controlling variable in many situations involving creep, and might also be expected to have an important influence in cyclic problems.

The strain-rate approach used in analyses of noncyclic situations presumes that the mechanical response of a material which behaves inelastically is defined by: inelastic strain-rate, inelastic strain, and stress. If the relationship between these three parameters is known for a material at a given temperature, it is theoretically possible to predict its flow behavior under any imposed conditions at that temperature. The monotonic relationship between strain-rate, strain, and stress at constant elevated temperature was presented in the form of a hodograph in Refs. 6 and 7, prepared from data obtained in stress-strain tests conducted under constant strain-rate to failure. The term hodograph, when used in mechanics of materials, describes a plot of strain-rate against strain with mechanical or thermal load as a parameter. Material flow behavior under a number of noncyclic loading paths was computed from the hodograph and compared with experimental data.

This paper describes an experimental study of the effect of changes in strain-rate on the relationship between strain and stress for Udimet 700 at 925C. An attempt was made to relate cyclic creep phenomena to the monotonic properties of the material in terms of instantaneous strain-rate. The investigation of Ref. 6 was extended to conditions of cyclic tension, in which rapid load-changes occur. Under cyclic conditions it is necessary to modify the hodograph in order to account for the effects of cyclic loading. The correspondence between the parameteric relationship under various monotonic conditions was presumed to hold under cyclic conditions as well, with simple modifications. Results of a number of cyclic tests in tension were compared with the monotonic hodograph and analyzed.

Additional tests on Udimet 700 and other materials are planned in the future, in order to substantiate further the trends observed during this initial study.

Hodographic Representation of Materials Behavior

Hodographic Development

In experimental investigations of materials properties one is usually concerned with the stress-strain-time relations. The influence of strain-rate has not been systematically studied. Thus the relationship between the stress-

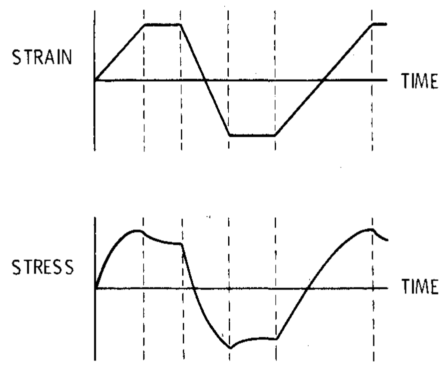

Fig.1 Typical low-cycle fatigue cycle at elevated temperature.

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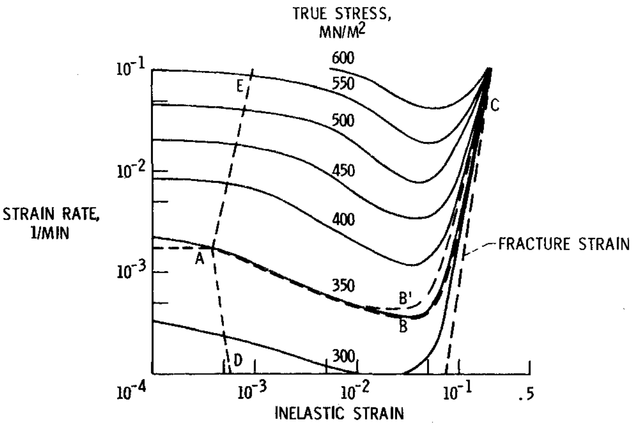

Fig. 2 Hodograph for U-700 in tension at 925C.

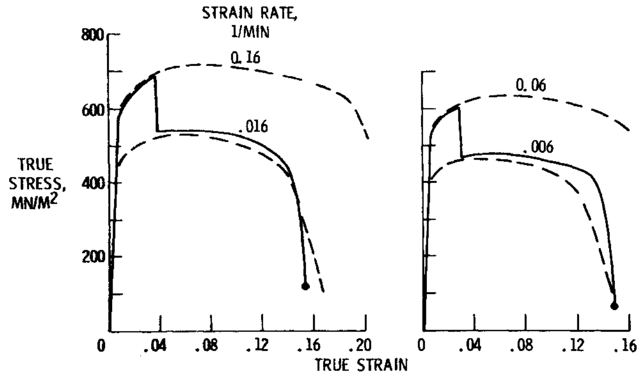

Fig. 3 Decrease in strain rate for U-700 at 925C.

strain test, the short-time relaxation test, and the long-time creep test is generally difficult to discern. However, it was shown in Ref. 6 that characteristics which these tests have in common may be clearly evaluated if materials data are plotted in hodographic form, as inelastic strain-rate against inelastic strain, with stress as a discrete parameter (Fig. 2). The hodograph is particularly useful because it presents the governing materials parameters, strain-rate and strain, as continuous variables without introducing the extraneous time term. At low values of strain the characteristic curves in Fig. 2 tend to be horizontal. This shows that for a given strain-rate there is a corresponding stress at which flow can be initiated. At high strains the characteristic curves become relatively steep. In between these extremes of strain the characteristics pass through a minimum, which indicates a maximum stress for a given constant strain-rate, or conversely a minimum strain-rate for a given constant stress.

The hodograph of Fig. 2 was constructed from true-stress true-strain data obtained at a constant strain-rate throughout.[6] This type of strength test permits stress to develop at a rate compatible with the imposed strain-rate condition and the characteristic rates of deformation processes of the material.

As inelastic strain is accumulated at a constant strain-rate, say 0.002/min, level with A, the true stress is observed to increase, pass through a maximum, and then decrease as failure is approached. The decrease in true stress indicates a creep-type failure. It was shown in Ref. 6 that the hodograph obtained from the stress-strain test also predicts other rate-controlled mechanical tests. Such tests include constant stress (ABC) and load (AB′C) creep, stress-relaxation (AD), and controlled reloading (AE), as well as combinations of these which do not require discontinuous changes in strain-rate.

Effect of Cyclic Loading Conditions

Unfortunately many cyclic-tension loading paths, such as depicted in Fig. 1, occur at rates which violate the bounds of the rate-controlled process as described above. Increase in stress is generally achieved at strain-rates higher than those characteristic of the lower stresses in the cycle, and often also higher than characteristic strain-rates at the maximum stress. Reversal of strain can only occur if compressive stresses are developed. Both of these situations involve discrete changes in strain-rate, and both are therefore incompatible with the conditions described by Fig. 2.

Modifications to the hodograph are required in order to account for the large changes in strain-rate which occur in cyclic problems. With the aid of monotonic tests modified by such changes, the required modifications to the hodograph can be determined.

Test Specimen, Equipment, and Procedure

Details concerning the type of test specimens, the low-cycle fatigue servo-controlled test facility, and general procedure have been adequately reported in Ref. 9. The form of specimen used in this investigation was the standard tubular hourglass shape shown in Fig. 1 (d) of Ref. 9. The Udimet 700 material of which the specimens were made was heat treated as follows: 1162C for 4 hr plus 1080C for 4 hr plus 843C for 4 hr plus 760C for 16 hr, with forced-air cooling after each phase. The material was tested at 925C, with a 30-min soak at temperature before application of load.

Stress-strain tests under varying strain-rates included a single increase or decrease of a factor of ten in strain-rate. Cyclic tensile creep tests were conducted under nominal maximum and minimum stresses of 450 MN/m^2 and 310 MN/m^2, respectively. Tests were performed with both high and low initial stress, and the strain-rates used during change in stress were 0.04/min. and 0.005/min.

Results and Discussion

Stress-Strain Tests with Strain-Rate Discontinuities

Results of stress-strain tests with a single, stepwise decrease or increase in strain-rate are shown as solid curves in Figs. 3 and 4, respectively. (These data are shown as true-stress-true-strain curves because the hodographic form in this case would mask the characteristics under discussion. The dashed curves represent corresponding constant strain-rate data.) Each test was initiated at a given strain-rate, and at some inelastic strain the rate of straining was increased or decreased ten-fold. After a decrease in strain-rate (Fig. 3) the material responded as if the lower strain-rate had been applied from the outset, and continued along the original stress-strain curve at this strain-rate. This behavior was to be expected because the reduction in stress which occurred immediately after the change in strain-rate was governed by conditions similar to those obtained during stress relaxation. It was shown in Ref. 6 that the more critical case of stress relaxation is compatible with the characteristic hodograph. The same should be true in the present case, therefore, when the strain-rate is reduced but still positive. Note that the prestrain at high strain-rate somewhat strengthens the material at large strains at the lower strain-rate, although ductility is slightly reduced.

The increase in stress developed after a discrete increase in strain-rate (Fig. 4) was insufficient to reach the stress-strain curve of the virgin material (without prestrain). When the change occurred in the primary or early secondary stage (approximately 4% inelastic strain or less), elastic stress-strain response recurred until the stress reached the proportional limit stress of the virgin material. At this point inelastic strain began to accumu-

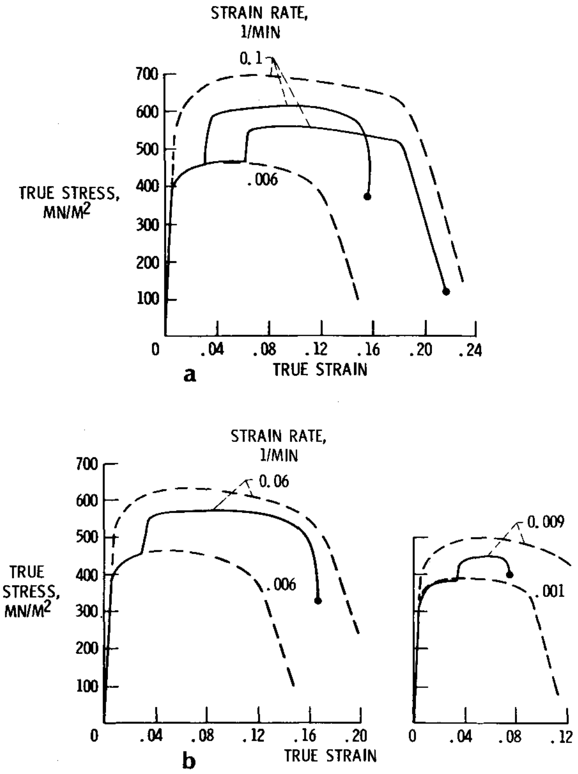

Fig. 4 Increase in strain rate for U-700 at 925C.

late again. The subsequent curves were rather flat, and little or no strain-hardening occurred. The stress remained in the region of the initial proportional limit stress until the fairly sudden drop-off before failure occurred. Total elongation was about 25% less than for the virgin material. When the material was prestrained beyond the point of maximum true stress (approximately 7% inelastic strain), the stress developed subsequently was lower than the initial proportional limit. This was probably due to the fact that the mechanism which would eventually lead to failure had already been initiated during prestraining at the low strain-rate.

These data indicate the manner in which the hodograph is altered if a discrete change in strain-rate occurs in a test. For a decrease in strain-rate the process is compatible with the hodograph as defined previously. For an increase in strain-rate the primary creep mechanism is reactivated, and the hodographic curve is raised. The constant-stress curve for the increased strain-rate starts at the level corresponding to the original flow stress for that strain-rate, but it does not dip downwards as strain is accumulated. Instead it remains essentially horizontal until third stage is reached, after which it again follows the original hodograph until failure occurs.

Prediction of Cyclic Tensile Creep

In the light of the material response to discrete changes in strain-rate, a simple method was devised for estimating time-dependent flow behavior under cyclic tensile loading conditions. The method is based on the following guidelines:

1. For loading that increases faster than prescribed by the hodograph, the stress and strain-rate reached are taken to be equal to the flow stress and corresponding primary strain-rate, as given by the hodograph. If either stress or strain-rate is maintained the other will remain at the corresponding primary-stage level until third-stage strains are developed. Thereafter the failure mechanism will operate as in a monotonic process.

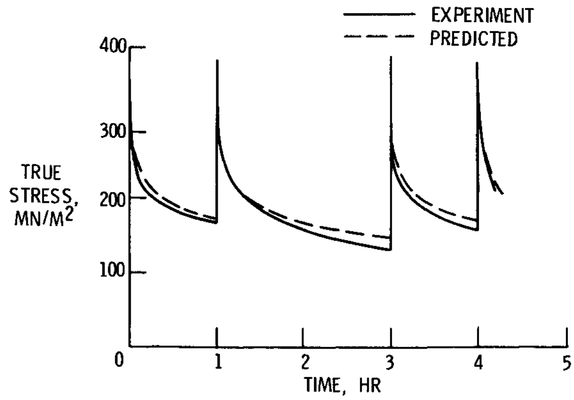

Fig. 5 Repeated stress-relaxation of U-700 at 925C.

2. In all other cases, with the exception of forced (compressive) unloading, the hodograph is taken to govern the material response.

Cyclic Creep Data

A number of cyclic tests was conducted in order to test the proposed method of calculation. Results of a repeated stress-relaxation test are presented in Fig. 5 and compared with predictions. The test included three reloads, which were carried out at high strain-rate so that negligible additional inelastic deformation occurred during reload; that is, reload occurred along a vertical path on the hodograph. Agreement between prediction and experiment in the three later cycles was almost as good as in the first cycle.

Four cyclic creep tests were performed, and the results are shown in Fig. 6 in hodographic form. The characteristic curves for the maximum and minimum stresses have been added for the sake of reference. The cyclic creep data are represented by dashed lines, and the modified hodographic line for the maximum stress used appears as dotted line. At the minimum stress the data lie close to the corresponding hodographic curve as predicted. The strain-rate at the maximum stress remained on the level of the primary strain-rate for that stress, until an inelastic strain of approximately 0.08 was accumulated, after which the data tended to follow the third stage portion of the characteristic curve. Thus the data obtained appear to substantiate the proposed method of prediction.

Cyclic creep lifetimes calculated for the tests are compared with experimental results in Fig. 7. Also included in the figure are results of monotonic creep tests reported previously (Ref. 6, circular symbols), as well as three tests (squares) consisting of stress-relaxation followed by rapid increase or decrease in stress and runout to failure at constant load. Cyclic creep results are shown as triangles. Although the present tests did not cover a large time-range, the good agreement shown in Fig. 7 is very promising.

Comparison with Other Theories for Cyclic Creep

A number of theories have been advanced in the past for describing creep behavior under cyclic loading, and subsequent to a sudden change in stress. Well known among them are the time-hardening, strain-hardening, and life-fraction rules.[10] These theories may be conveniently compared with the method proposed here with the aid of the hodographs in Fig. 8.

According to the time-hardening rule, the creep behavior of a material at any stress level depends on the value of stress and the total time elapsed since the beginning of the creep process. Change of stress is achieved along lines of constant time, which are represented at 45° to the hori-

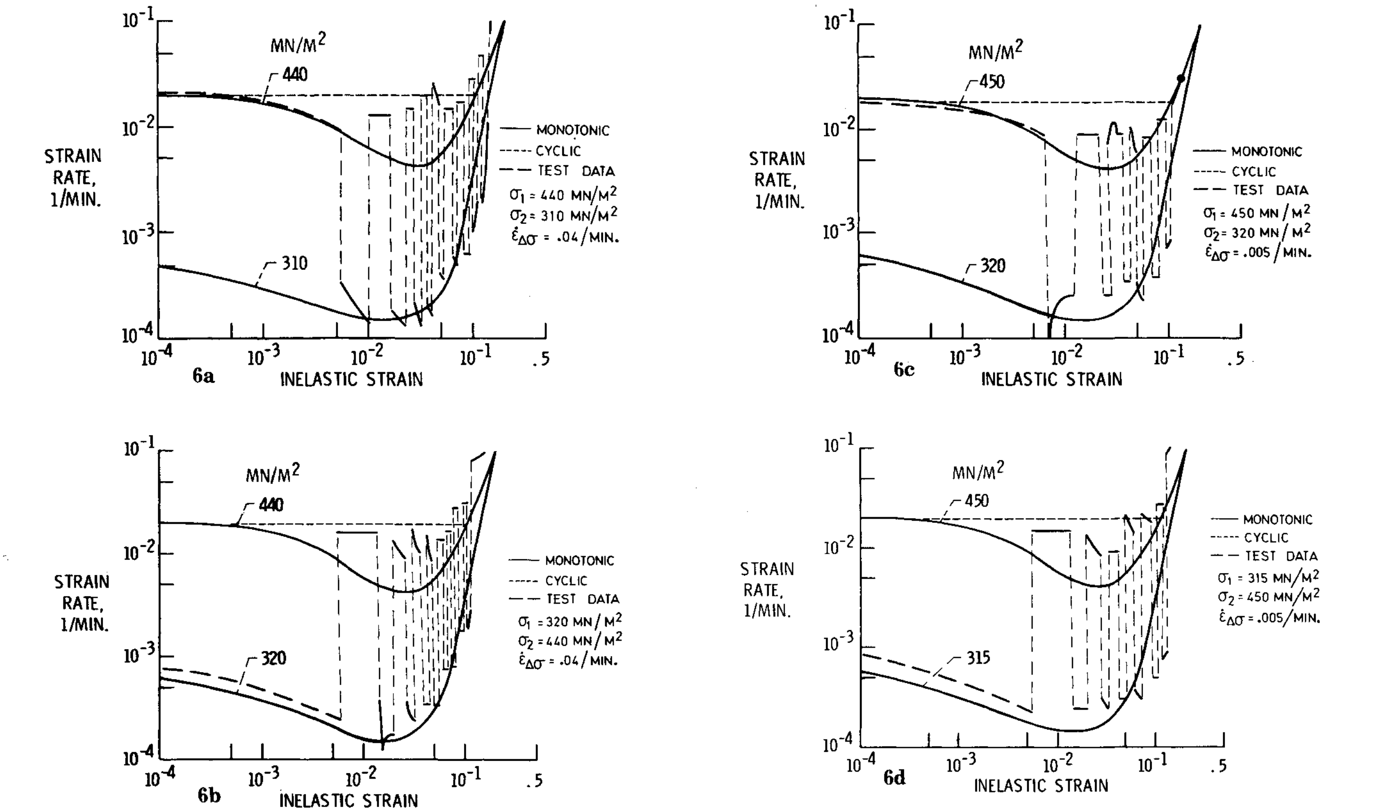

Fig. 6 Cyclic tensile creep of U-700 at 925C.

zontal on the hodograph (AB′ in Fig. 8). A rapid increase in stress at point A in Fig. 8(a) will result in a moderate rise in strain-rate to point B′. The material will continue to creep along the curve designated "time-hardening" in the figure, from the point B defined by the creep strain at A and the strain-rate at B′. A rapid decrease in stress will result in a moderate drop in strain-rate from point A in Fig. 8b to point B′. The subsequent creep curve begins at B, Fig. 8b, defined as before. Note that once the end of the so-called secondary stage has been reached, a stress-increase apparently cannot be accomplished along a line of constant time, since the hodograph indicates decreasing stresses in this region.

The strain-hardening rule states that material responds to a change in creep stress as if the change were achieved at constant inelastic strain. It does not take into account the effect of strain-rates obtaining during the stress change. The strain-hardening rule is represented in Fig. 8a, 8b by the vertical line AC. The creep curve follows the hodographic curve for the new stress from point C, whether the stress was increased or decreased.

From Fig. 8 it is clear that the strain-hardening rule predicts higher strain-rates than does the time-hardening rule when stress is increased in the primary stage, or decreased in the third stage. The strain-hardening rule predicts lower strain-rates when stress is decreased in the primary stage, or increased in the third stage. If the applied loading is cycled a number of times before failure occurs, these opposing effects will tend to cancel each other, and predictions made in accordance with the time-hardening and strain-hardening rules will converge on each other.

The life-fraction rule states that the damage occurring in a material is a function of the fraction of rupture time which has been consumed. Although the life-fraction rule was conceived as a criterion for rupture life, it has been applied to the calculation of flow behavior. In the interest of clarity the life-fraction rule is not depicted in Fig. 8. However, it generally falls between the time-hardening and strain-hardening rules on the line BC, and proceeds between the two dashed curves. It has been shown[11] that the life-fraction and strain-hardening rules coincide when the total

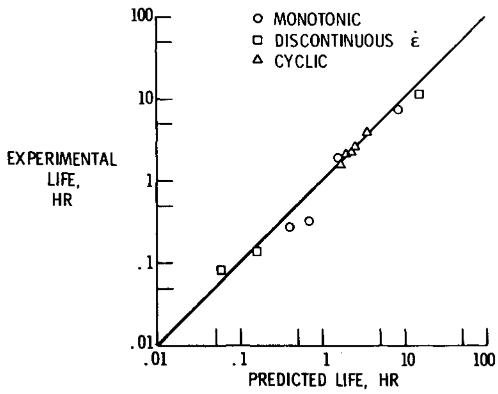

Fig. 7 Creep failure of U-700 at 925C.

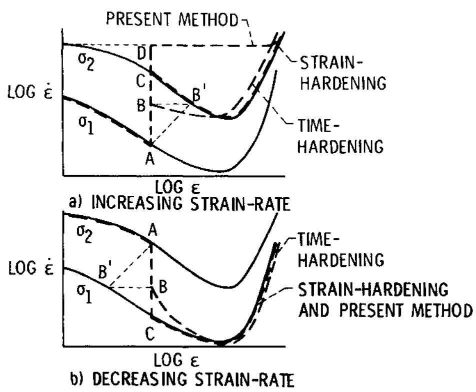

Fig. 8 Comparison between cyclic creep theories.

elongation at fracture is constant and independent of strain-rate. Under this condition a given inelastic strain (strain-hardening rule) corresponds to a fraction of rupture time which is constant at any level of strain-rate or stress (life-fraction). The equal-damage concept proposed in Ref. 6 is vertical in this case, coincident with the strain-hardening and life-fraction rules.

The method suggested herein for predicting cyclic creep flow is also represented in Fig. 8. For reduction in stress the proposed method coincides with the strain-hardening rule. For an increase in stress the method predicts a strain-rate which is considerably higher than that predicted by the other theories discussed. In the latter case the resulting failure time can be from 3 to 8 times less than predicted by the other theories.

Summary of Results

A study was made of the significance of strain-rate in the prediction of inelastic properties of Udimet 700 under cyclic tension at 925C. Stress-strain tests were conducted with step-wise changes in strain-rate, to determine the nature of modifications caused to the hodograph obtained under monotonic loading. A straightforward method, based on the hodograph, was suggested for predicting cyclic creep behavior, and results calculated by this method were compared with test data obtained.

The major results of the investigation are as follows:

1. Continuous increases and all decreases in strain-rate result in behavior which corresponds to the characteristic hodograph. When discontinuous increase in strain-rate occurs, primary creep is reactivated until the inelastic strain reaches third-stage magnitude.

2. A simple method is proposed for estimating creep response under cyclic tensile stresses from the hodograph. Comparison between predicted response and results of cyclic creep tests in tension show good agreement.

3. In the light of present results the proposed hodographic method compares favorably with previously published theories of cyclic creep. However judgement must be reserved until further data on other materials and at other temperatures become available.

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